# LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – STATISTICS

THIRD SEMESTER – APRIL 2010

**ST 3503/ST 3501/ST 3500 - STATISTICAL MATHEMATICS - II**

Date & Time: 23/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

## PART – A

Answer **ALL** Questions. (10 x 2 = 20 Marks)

1. What do you mean by a subdivision of a closed bounded interval [a, b]?
2. Find the M.G.F. of the random variable X having p.d.f :

$$f(x)=\begin{cases} x, & for\, 0\le x<1 \\ 2-x, & for\, 1\le x<2 \\ 0, & otherwise \end{cases}$$

3. Prove that $\Gamma(n+1)=n\Gamma(n)$.
4. Find L ($\cos^2 3t$).
5. Evaluate $\int_0^2\int_0^1 (xy)dxdy$
6. Find the Cov (x,y) of the bivariate distribution $f(x,y)=\begin{cases} 1, & 0<x,y<1 \\ 0, & otherwise \end{cases}$.
7. Solve the equation $\frac{d^2y}{dx^2}+5\frac{dy}{dx}+7y=0$
8. Define a Poisson process.
9. State Cayley – Hamilton theorem on matrices.
10. Find the characteristic roots of the matrix $A=\begin{pmatrix} 2 & 5 \\ 8 & 7 \end{pmatrix}$.

## PART – B

Answer any **FIVE** questions (5 x 8 = 40 marks)

11. Check whether the function f:[0,1] $\to R^1$ such that

$$f(x)=\begin{cases} 1, & When\ x\ is\ a\ rational\ number \\ 0, & When\ x\ is\ an\ irrational\ number \end{cases}$$

is Riemann integrable.

12. Evaluate $\int_0^1 (x\log x)^4 dx$.

13. Let X be a continuous random variable with p.d.f:

$$f(x)=\begin{cases} \frac{x}{2}, & o\le x<1 \\ \frac{1}{2}, & 1\le x<2 \\ -\frac{x}{2}+\frac{3}{2}, & 2\le x\le 3 \\ 0, & otherwise \end{cases}$$

Find the mean and variance of X.

14. Prove that the improper integral $\int_1^\infty \frac{dx}{x}\, diverges$.

**(P.T.O.)**

15. Find $L^{-1}\left(\frac{s^2}{(s-2)^2}\right)$.

16. Evaluate $\int\int e^{3x+4y}\,dx\,dy$ over the triangle bounded by x = 0, y=0, x+y=1.

17. Solve the equation $(D^2 - 1)$ y= x sinx.

18. Verify Cayley – Hamilton theorem for the matrix $\begin{bmatrix} 1 & 0 & 3 \\ 2 & 2 & -1 \\ 1 & -1 & 1 \end{bmatrix}$.

## PART – C

Answer any **TWO** Questions (2 x 20 = 40 marks)

19. a) State and prove second fundamental theorem of integral calculus.

b) A random variable X has the p.d.f $f(x) = \begin{cases} |x|, & -1 < x < 1 \\ 0, & otherwise \end{cases}$

Find the first four onoments of the random variable about its mean.

20. a) Prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ .

b) Prove that $\int_1^\infty \frac{dx}{x^2}$ converges and $\int_1^\infty \frac{dx}{\sqrt{x}}$ diverges.

21. a) Let (X,Y) be a two dimensional continuous r.v. having the joint density.

$$f(x,y) = \begin{cases} 4xy\, e^{-(x^2+y^2)}, & x>0, y>0 \\ 0, & otherwise \end{cases}$$

Find the density function of $u = \sqrt{x^2 + y^2}$ .

b) A continuous random variable X has the following p.d.f.

$f(x) = a + bx,\ 0 \le x \le 2$.

If the mean of the distribution is 1 find the values of a and b.

22. a) Using Laplace transform, solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{-x}$ $given\ y = 0$

$\frac{dy}{dx} = 1\, when\, x = 0$.

b) Find whether the following equation are consistent and if so, find the solution.

$x + 2y + 2z = 2$ ; $3x - 2y - z = 5$; $2x - 5y + 3z = -4$;

$x + 4y + 6z = 0$

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